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EXPERIMENTAL MODAL ANALYSIS OF A CAR WINDSHIELD

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Abstract: The automakers are continuously interested in noise vibration and harshness solutions for high quality vehicles in general and quieter passenger cabins in special. In this paper the modal parameters of a car windshield are determined by using experimental modal analysis. The frequency response functions have been measured by using impulsive technique. An impact hammer has been used to generate the impulsive force applied at various locations evenly spread on the windshield surface. A fixed mini-accelerometer has been used to measure the structure response. Starting from the set of FRF functions of free-free conditions, the Least Square Complex Exponential parameter estimation method implemented in LMS Test.Lab software has been used to extract the modal parameters. The validation phase sustained the correctness of the modal parameters found. **Key words:** experimental modal analysis, frequency response function, impulse response, modal parameters, windshield.

1. INTRODUCTION

The automakers are continuously interested in Noise Vibration and Harshness (NVH) solutions for high quality vehicles. Hence, ways to reduce structure-born noise, road and wind noise entering the passenger compartment are observed.

Managing the physical characteristics of the car windshield is important in the quest for quieter passenger cabin, for offering high quality acoustic environment for occupants, in the effort of manufacturing better automotive components and to offer occupant safety and pedestrian protection. A new demand for quieter vehicles is to facilitate the use of telematics within the cabin, including voice-activated technology and mobile phones. The dynamic range of speech is about of 30dB for each third octave band from 200 to 6000 Hz. Reducing noise contributes to increasing speech intelligibility as well. Acoustic energy can be easily transmitted through the vehicle windshield comparing with the rest of the interface to the exterior. When running with high speed, air pressure variations are registered at the windshield level causing glass to radiate noise in the compartment.

In the present study a standard laminated glass windshield with two layers of glass and an inner layer of polymer is considered. The inner layer, often an elastomeric polymer called polyvinyl butyral (PVB), is pressed under heat, between the two sheets of glass. The Young modulus of the PVB is much lower than that of the glass. In the PVB viscoelastic layer an important transverse shear is induced. This transverse shear introduces damping. The PVB layer damps vibrations, prevents parts of the glass to spread around and offers retention to the occupant in case of accident [4].

Regarding the structure, one can mention acoustic windshields having five layers: two exterior layers of glass, two intermediate layers of polymer and one inner layer of an acoustic polymer [2]. The inner layer has the function of damping frequencies specific to the acoustic range, hence protecting the vehicle cabin.

2. EXPERIMENTAL MODAL ANALYSIS

2.1 General aspects

Experimental modal analysis (EMA) is the process of extracting modal parameters of a mechanical structure starting from vibration data measured on the structure. When starting

from frequency response functions (FRFs) the extraction process is called frequency response domain analysis. FRF is a description of the input-output relationship as a function of frequency between two degrees of freedom of the structure. When the impulse responses (IRs) are used, the approach is called time domain modal analysis. The mathematical model in case of frequency domain modal analysis is an analytical expression of a FRF best fitted with the measured FRFs. In reality a structure has an infinite number of DOF and poles.

Lightly damped modes have narrow peaks in frequency domain described by a few points, hence the modal parameters are not so easy to be determined. On the contrary, in time domain, the signal of the impulse response has long duration decay and is easier to be identified.

Over the last years, techniques for parameter identification only from output data have been developed. One can mention Auto-regressive moving parameters (ARMA), Natural extraction techniques (NexT) or stochastic realization methods.

2.2 Least-square complex exponential (LSCE) parameter estimation method

The LSCE approach will be shortly recall [3]. LSCE method is a time domain approach which uses the impulse response functions IRFs of a multi degree of freedom system to estimate the complex poles and residues. The measured set of FRFs are expressed by relations (1).

$$H_{ij}(s) = \sum_{r=1}^n \left(\frac{r A_{ij}}{s - s_r} + \frac{r A_{ij}^*}{s - s_r^*} \right) \quad \text{or} \quad (1)$$

$$H_{ij}(s) = \sum_{r=1}^{2n} \frac{r A_{ij}}{s - s_r} \quad \text{for } r > n$$

These relations are transformed into the IRFs by inverse Laplace transform, resulting:

$$h_{ij}(t) = \sum_{r=1}^{2n} r A_{ij} e^{s_r t} \quad (2)$$

IRFs are sampled at equally spaced time intervals dt :

$$h_{ij}(k \cdot dt) = \sum_{r=1}^{2n} r A_{ij} e^{s_r \cdot k \cdot dt} \quad (k = 0, 1, \dots, 2n) \quad (3)$$

After the substitution:

$$z_r^k = e^{s_r \cdot k \cdot dt} \quad (4)$$

we obtain:

$$h_k = \sum_{r=1}^{2n} r A_{ij} \cdot z_r^k \quad (k = 0, 1, \dots, 2n) \quad (5)$$

where $h_k = h_{ij}(k \cdot dt)$ are values known from measurements (real values), A are the residues (complex values) and z are complex values because the poles s are complex.

Considering that z are roots of a polynomial with real coefficients, we obtain:

$$\sum_{r=0}^{2n} \beta_k \cdot z_r^k = 0 \quad (6)$$

Using the available data from IRFs (measured data) we are able to estimate the coefficients β found in relation (6). In relation (5) multiplying each equation with a specific coefficient β and adding all equalities together, yields (7).

$$\sum_{k=0}^{2n} \beta_k h_k = \sum_{k=0}^{2n} \beta_k \sum_{r=1}^{2n} r A_{ij} \cdot z_r^k \quad (7)$$

We can interchange the summations:

$$\sum_{k=0}^{2n} \beta_k h_k = \sum_{r=1}^{2n} r A_{ij} \cdot \sum_{k=0}^{2n} \beta_k \cdot z_r^k \quad (8)$$

In the right side of the equation we have a summation which is zero when z_r is a root of the polynomial mentioned in relation (6). For these z values, the right member of the equation is null, resulting:

$$\sum_{k=0}^{2n} \beta_k h_k = 0 \quad (9)$$

By using the relation (9) and $2n$ sets of $2n$ samples of the known IRFs, we can estimate the set of β coefficients. Dividing by β_{2n} we obtain the value 1 for the coefficient of h , placed on the right side of each equation.

We get $2n$ linear equations from which the coefficients β_i can be estimated:

$$\begin{bmatrix} h_0 & h_1 & h_2 & \dots & h_{2n-1} \\ h_1 & h_2 & h_3 & \dots & h_{2n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ h_{2n-1} & h_{2n} & h_{2n+1} & \dots & h_{4n-2} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{2n-1} \end{bmatrix} = \begin{bmatrix} h_{2n} \\ h_{2n+1} \\ \vdots \\ h_{4n-1} \end{bmatrix} \quad (10)$$

For the least square solution the number of equations (rows in the matrix equation) can be greater than the number of unknown coefficients.

The data samples h_i , $i = 0, \dots, 4n-1$ in each row are evenly spaced in time.

Once the β coefficients are known we can solve the equations (6) for the unknown z roots:

$$\sum_{r=0}^{2n} \beta_k \cdot z_r^k = 0$$

From these and considering the expressions of the poles:

$$\begin{aligned} s_r &= -\zeta_r \omega_r + j\sqrt{1-\zeta_r^2} \omega_r \\ s_r^* &= -\zeta_r \omega_r - j\sqrt{1-\zeta_r^2} \omega_r \end{aligned} \quad (11)$$

can be extracted the natural frequency and damping ratio for each mode (12):

$$\omega_r = \frac{1}{dt} \sqrt{\ln z_r \ln z_r^*}, \quad \zeta_r = \frac{-\ln(z_r z_r^*)}{2\omega_r \cdot dt} \quad (12)$$

The derivation of the system mode shapes resides in finding residues A :

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ z_1 & z_2 & z_3 & \dots & z_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ z_1^{2n-1} & z_2^{2n-1} & z_3^{2n-1} & \dots & z_{2n}^{2n-1} \end{bmatrix} \begin{bmatrix} 1 A_{ij} \\ 2 A_{ij} \\ \vdots \\ 2n A_{ij} \end{bmatrix} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{2n-1} \end{bmatrix} \quad (13)$$

Further details of the LSCE method can be found in references [1] and [3].

2.3. Measurement set-up

The windshield has been suspended on elastic cords trying to minimize the stress in the structure and to allow the rigid body modes movement.

Experimental modal analysis by using impact testing has become largely known as a fast and economical mean of finding modal parameters of a structure.

For the FRF measurement, the impulsive method has been used assuming the symmetry of FRF matrix. The impact hammer was connected to the first channel of an LMS acquisition system and was used to excite the structure in 47 locations (roving hammer).

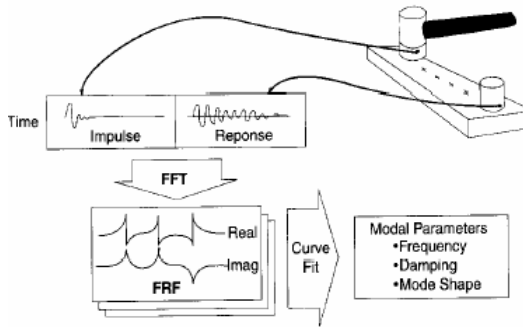


Fig. 1. Experimental set-up

A PCB mini-accelerometer of 1.5 g placed into the reference point is connected to the second

channel of the acquisition system in order to record the response of the structure. A column of 47 FRFs of the transfer matrix has been built. The schematic overview of the experimental test set-up and FRF derivation is shown in figure 1.

2.4. Modal parameters estimation

For the parameter estimation the LMS Test-Lab dedicated software has been used. The identification process is starting from a set of 47 FRFs measured on the windshield. The impact locations spread on the windshield surface can be seen in figure 2.

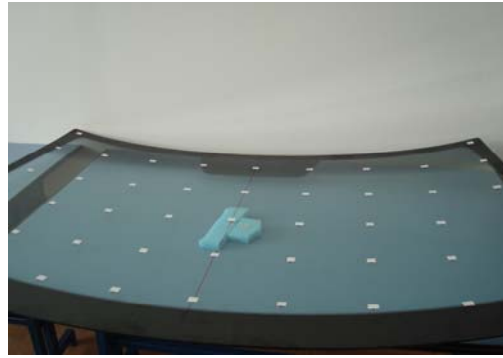


Fig. 2. Impact locations on the windshield

When assessing the poles, the frequency band of analysis should not be very wide. It is advisable to select several consecutive narrow bands instead of a large one in order to evaluate the parameters of the band of interest. When the estimation approach is based on a time domain technique and we are starting from FRFs, each frequency band is advisable to contain a power of two frequency lines. This will ease the conversion of data from frequency domain to time domain. In order to minimize the out of band effects the FRFs values at the limits of the band should be small (choosing local minima).

The estimation of the physical pole number can be done by counting the peaks in the FRF amplitude function. A frequently used method is to visualize the SUM of the measured FRFs (amplitude) and to count the peaks. Large peaks indicate large total modal displacements, while small peaks indicate small total displacements or local modes of vibration. In general the number of peaks will be less than the roots number, one cause being the fact that a multiple root is associated to one peak.

The Mode indicator function (MIF) can help in counting the poles. The local minima of the MIFs indicate the resonances of the normal modes of the structure.

The global parameter stabilization diagram for the first frequency band is shown in figure 3. This diagram is a good tool to find the physical poles.

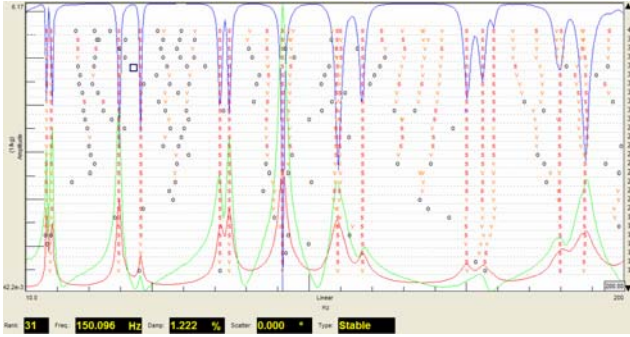


Fig. 3. Stabilization diagram

An important aspect is to select a correct model order. Some of the resulting eigenvalues do not have a physical meaning. These eigenvalues are in general caused by mathematical effects or noise and result from the forced fulfillment of the estimated model order. The Stabilization diagram is used to bring together the poles coming from successive analysis, each analysis with a different model order. The horizontal axis of the diagram registers the pole frequencies and the vertical axis the solution order. A pole in the diagram is indicated by a specific character (like s for stable pole).

After the pole selection is completed, the modal shapes are calculated by using the complex residues [9]. The first 14 complex poles, damping and damped frequencies are listed in Table 1.

A simplified geometry, created in order to visualize the mode shapes, can be seen in figure 4.



Fig. 4. Windshield representation

In figures 4,5,6 and 7, the first eight mode shapes are depicted and grouped in pairs. The

first mode is the first bending of the structure, the second is a twist and so on.

Table 1

Complex poles				
Mode #	Pole Imag	Pole Real	Damping (%)	Frequency (Hz)
1	103.945	-0.824	0.79	16.54
2	113.274	-2.158	1.90	18.02
3	247.991	-3.220	1.29	39.46
4	291.63	-2.698	0.92	46.41
5	450.528	-5.810	1.28	71.70
6	469.043	-4.844	1.03	74.65
7	575.234	-6.815	1.18	91.55
8	684.978	-10.91	1.59	109.0
9	734.834	-9.198	1.25	116.9
10	943.156	-11.60	1.22	150.1
11	975.039	-11.63	1.19	155.1
12	996.846	-6.456	0.64	158.6
13	1129.96	-16.33	1.44	179.8
14	1181.59	-17.30	1.46	188.0

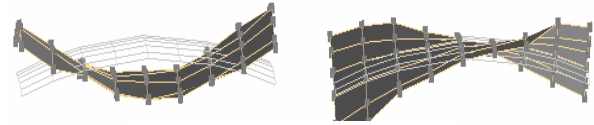


Fig. 5. Modes #1 and #2

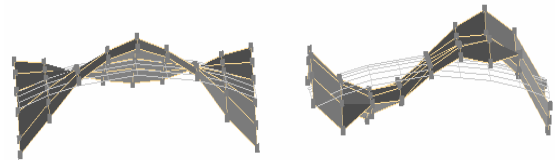


Fig. 6. Modes #3 and #4

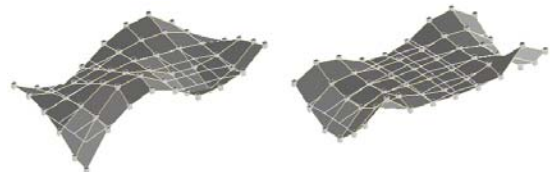


Fig. 7. Mode shapes #5 and #6

A good correlation has been ascertained when superposing a measured and the associated synthesized frequency response function. A typical pair of measured FRF versus synthesized FRF is shown in figure 8.

Our estimation of the poles is observing the measured frequency band, hence many poles will be neglected. Each measured FRF is

containing modal information of all modes of vibration including the modes not belonging to the frequency range of measurements. The difference between the measured FRFs and the estimated FRFs is the error function. We are trying to identify the modal parameters of the structure by minimizing the error function.

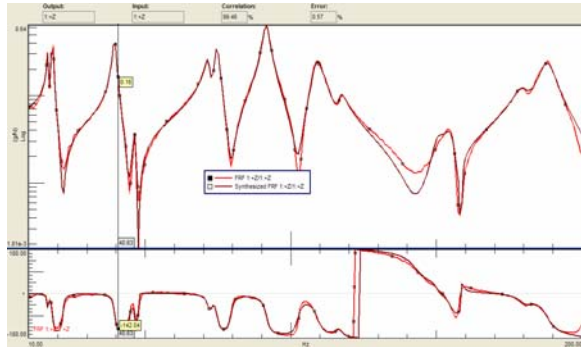


Fig. 8. Measured and synthesized FRF

A series of parameters, such as Modal scale factors (MSC), Modal assurance criterion (MAC), Mode complexity, Modal phase colinearity (MPC), Mean phase deviation (MPD), were used to validate the accuracy of the modal model (frequencies, damping values, mode shapes and participation factors).

The MAC matrix is depicted in figure 9.

The complex modal vectors are linear independent since the off diagonal MAC values

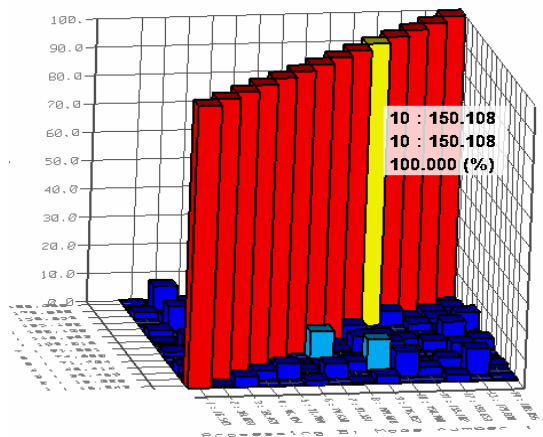


Fig. 9. The MAC matrix

are small. MPC, MPD, and Mode participation (MP) are listed for each mode of vibration in Table 2. From MPC and MPD we conclude that the modes are lightly damped and almost real.

The scatter parameter is low for all first 14 modes. The modal participation of different

modes in the analysis frequency band can be found in column MP of the table.

Table 2

Validation parameters				
Mode #	Frequency (Hz)	MPC (%)	MPD (°)	MP (%)
1	16.54	98,95	6,07	22,97
2	18.02	99,90	1,93	34,65
3	39.46	99,96	1,01	13,38
4	46.41	99,75	2,87	3,11
5	71.70	99,93	1,56	4,71
6	74.65	99,94	1,35	4,57
7	91.55	99,93	1,45	6,33
8	109.01	99,84	2,28	4,18
9	116.95	99,9	1,88	1,42
10	150.10	99,20	5,40	0,77
11	155.18	98,56	7,18	0,72
12	158.65	97,28	10,20	0,16
13	179.83	97,39	9,68	1,15
14	188.05	98,69	6,95	1,82

3. CONCLUSION

The modal model described by the damped natural frequencies, the damping values, mode shapes and participation factors has been determined for a standard laminated glass windshield with two layers of glass and one PVB viscoelastic layer. The identification technique has started from the FRFs measurement on the structure on free-free conditions, by using the impulsive technique and a roving hammer. The impulse response functions (time domain) are calculated and the modal parameters are estimated by using the Least Square Complex Exponential method. A set of specific parameters were used in order to validate the identified dynamical parameters of the structure.

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Analiză modală experimentală aplicată unui parbriz de automobil.

Rezumat: În lucrare sunt determinați parametrii modali ai unui parbriz de automobil prin analiză modală experimentală. Funcțiile de răspuns în frecvență au fost măsurate folosind tehnica impulsului. S-a ales o rețea de locații uniform distribuite pe suprafața parbrizului pentru aplicarea forței impulsive cu ajutorul unui ciocan prevăzut cu senzor de forță. Un miniaccelerometru a înregistrat simultan răspunsul la impact. Parametrii modali ai structurii (frecvențele de rezonanță, formele de vibrație și amortizările modale) au fost evaluați pe baza funcțiilor de răspuns în frecvență măsurate în condiții de suspendare elastică a structurii și folosind metoda LSCE implementată în produsul LMS Test.Lab. Rezultatele obținute au fost validate pe baza unui set de parametri specifici.

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